Learning Goals

In this lab, students will engage with taking direct and indirect measurements, examining collected data to understand accuracy and precision, and performing remote distance calculations similar to stellar parallax distance determinations. In this lab, students will

• Perform simple scientific measurements
• Carefully record data
• Examine the distribution of data points over several different variables
• Explore the concepts of accuracy, precision, and uncertainty
• Perform remote distance calculations
• Play a game

Materials

• Game board, bean bags, direct distance measurement tools, and devices to measure angles, including a protractor.
• Prepared Google response form:

1. Background

1.1 Introduction to Scientific Measurement

How do we go about determining things like the number of galaxies in the universe, how much “stuff” the universe contains, the value of the gravitational constant of the universe, and the distance to the stars or how bright they are? Where do we get the evidence to evaluate some of our biggest astronomical questions, such as how did the universe, the solar system, and Earth get their start? To answer all of these questions, scientist must collect observations and empirical evidence to test the predictions of their hypotheses and theories against. The collection of these observation and evidence almost certainly involves measuring some quantity of something. Whether that be the amount of light being received by a distant supernova, the total amount of mass (both luminous and dark) in a galaxy cluster, or the distance to the stars of the Milky Way, observations must be made, and data must be collected. How well we can measure the related quantities is intimately tied to how well we can evaluate our deepest scientific understandings of the universe.

Answering some of the above questions is a daunting task, and often times we only have a ballpark idea of what the answer might be. In more intriguing cases, we find ourselves in completely new scientific territory where we have little theory to rely upon. Here, science must tackle challenging questions in face of the unknown. There may be no known methods from which we can start trying to solve the problem. We are left to rely on our own creativity and ingenuity to develop new problem-solving techniques. If we are clever, we often can devise multiple ways to address a question. Some of the methods will provide more reliable
answers than others. With hypothesis, theory, and ingenuity as our resources, we then have to determine, using the principles of science, which methods are best. This frequently comes down to how those methods make their measurements, what they are measuring, and which one gives the explanations we have the most confidence in. This means we need to understand the empirical evidence, frequently just called data in science, itself. How reliably does it give an answer close to the correct one, and how much does the data vary from measurement to measurement?

In this lab, we will explore the some of the techniques scientists and astronomers use to begin addressing how to approach scientific problems and gather the required evidence. We will explore the difficulty in making measurements, the uncertainty and errors associated with measurement, and how we can compare measurements of the same thing made with different tools or techniques.

1.2 Scientific Measurement

Measurement often is thought of as a simple task. For most people, daily life measurements are simply measuring how long something is with a measuring stick, how much volume you need with a measuring cup, the mass of something with a scale or a mass balance, or how long something takes with a stopwatch. If you are off by a few centimeters, milliliters, grams, or seconds, then so be it, there was no real harm done. In science, however, we strive to arrive at the real value as close as possible, or rather, to have measurements with high levels of **accuracy**. To have an **accurate** measurement, or set of measurements, is to be close to the real value with the measurement, or the average of the set of measurements. To accomplish accurate measurements, much more care and sophisticated instruments are needed than you experience in everyday life.

In science, we also never rely upon a single measurement. Science demands that we make measurements repeatedly to check and re-check our evidence. In these repeated trials, scientist aim for measurement methods that consistently give values that are nearly the same, or rather, have high levels of **precision**. How close they are together, or how precise your measurements are, determines how certain we can be that our evidence provides a good test for the predictions of our hypotheses and theories. In other words, the precision of our measurements quantifies the **uncertainty** in our measurements, or rather, what is the probability that our measurement is accurate within a certain range of values. A graphical representation of these concepts is provided in Figure 1.

![Figure 1. A graphical representation of accuracy and precision. For repeated measurements, the accuracy is how far the average of the measurements is from a reference value, represented by the vertical line through the measurement distribution. The precision is the spread of all measurements. A quantification of the width of the distribution of measurements would be the uncertainty.](image)
Altogether, scientists strive to measure as close to the true value as possible (accuracy) and repeatedly get measurements that are near to the same value (precision), which in turn, allows them to understand, with statistical confidence, they have homed in on the true value plus or minus a little bit (uncertainty). It should be noted that a real, or true, measured value\(^1\) is an ideal concept. It exists, but it is impossible to measure exactly.

1.2.1 Direct versus Indirect Measurement

The examples of measuring distance, volume, mass, and time given above are all examples of direct measurement. This is the simplest and most ideal case of the quality of your measurement coming down to the precision of your measuring instrument. For example, if you have a meter stick that marks centimeters and millimeters, you can measure down to the millimeter plus or minus a half millimeter. The plus or minus half a millimeter (+/- 0.5 mm) is the uncertainty of the measuring instrument. A general rule of thumb in measurement is that the uncertainty of an instrument is half the smallest division on the instrument. Another example is measuring the mass of something on a digital scale. If the digital scale gives the mass in grams to one decimal place, then the uncertainty of any single measurement is +/- 0.05 grams.

In many scientific fields, and especially in astronomy, scientists do not have the luxury of making direct measurements. Instead, they must rely upon indirect measurement, which involve techniques that exploit a quantity’s dependency (often mathematical) on something that can be directly measured. For example, astronomical objects (e.g., the Moon, planets, stars, galaxies, etc.) are physically too far away to measure directly, so astronomers must rely on indirect techniques to measure their distances. Measuring distances accurately and precisely turns out to be perhaps the most difficult task astronomer’s face! A few examples that you will explore in the lab activities and lab questions of this are the techniques to determine the distances to stars and planets.

1.2.2 Astronomy Indirect Measurement Example 1: Distance from radar methods

The distances to planets (and the Moon) are made using radar (RAdio Detection And Ranging). In this indirect distance measurement, astronomers time how long it takes the radio wave (a type of light) to travel to the object, reflect off it, and travel back to Earth. The radio wave travels at the speed of light (3.0 x 10\(^8\) meters per second), so how long the journey takes can be mathematically translated into a distance via

\[
Distance = velocity \times time
\]

Where the velocity is the speed of light, \(c=3.0\times10^8\) m/s and the time is the measurement made. In indirect measurements the accuracy, precision, and uncertainty of the desired quantity depends on the accuracy, precision, and uncertainty of what is measured. In the case above, the uncertainty in the time measurement must be translated through to get the uncertainty in the distance. This translation of uncertainty from the direct measurement to the indirect one is called propagation of uncertainty (sometimes propagation of error).

\(^1\) As opposed to a counted value, which has a distinct number. In the case of counting very large numbers, however, it may be intractable to determine a true countable number.
1.2.3 Astronomy Indirect Measurement Example 2: Distance from stellar parallax

In the other example, astronomers use the technique of stellar parallax to measure the distances to stars. Stellar parallax is a method to determine distance via triangulation. **Triangulation** is a method to determine the distances between two points through drawing series of (right) triangles, where the distance of one side of the triangle can be directly measured, and at least one of the two non-right angles can be directly measured. With a known distance for one of the sides of the triangle, called the baseline distance, and a measured angle of a right triangle, it is possible to calculate the distance of another side of the triangle, which in this case, is the distance of a far-off object.

Astronomers use a similar technique, but the angle measured is determined by the phenomenon known as parallax, which is the angular displacement an object appears to have against a fixed background when viewed from two different positions. Think about holding your thumb out at arm’s length and looking at it by blinking between using only our right and left eye. You thumb appears to shift with respect to the background. When applied to the stars, this apparent angular shift is referred to as **stellar parallax**. The general method of stellar parallax is shown in Fig. 2. With parallax, the baseline is the distance between the two viewing vantage points. Since, the stars are so very far away, astronomers need to use the largest baseline possible in order to actually measure an angle. For this, astronomers use the diameter of Earth’s orbit as the baseline distance because it is the largest baseline, a full 2 AU, accessible to humans! The angular displacement is the apparent angular shift a star appears to have when viewed from opposite sides of Earth’s orbit. The displacement angle is measured directly, and used to indirectly determine the distance. Half of that measured displacement is called the parallax angle, $p$, which often is just referred as the stellar parallax [See Fig. 2]. It can be translated into distance using a relatively simple equation:

$$\text{Distance [pc]} = \frac{1}{p[\text{arcsec}]}$$  \hspace{1cm} (2)

Since the stars are very far way, the stellar parallax angle is very small and hard to measure. This makes it prone to high levels of uncertainty, which propagate to a large uncertainty in distance. In astronomy, measuring distances is extraordinarily difficult!

![Diagram of stellar parallax](image)

*Figure 2. The method of stellar parallax to determine the distances to nearby stars.*
1.3 The Difficulty of a Scientific Measurement – A Direct Measurement Example

Scientific measurement is a deceptively difficult task with a large amount of nuance. Consider the simple task of measuring exactly how long a room is. For simplicity, let us imagine that the room’s floor print is a square, so that the length and width are equal. How would you go about measuring the length of the room? Likely, you would get a measuring tape, a meter/yard stick, or some other standard of length measurement and just see how long the room is. Since we are working in the world of science, let us imagine that you chose to measure the length in meters. Unfortunately, you only have a meter stick in your room, but nevertheless, it seems like a good one with centimeters (cm) and millimeters (mm) clearly marked. So, you set to the task with confidence that you can determine the length of the room. You measure 5 full meters and have a bit leftover where the meter stick is too long to measure the remaining distance to the wall. Being clever, you mark the 5-meter mark on the floor and then measure from the wall out to that point to and find that length to be 35.6 centimeters. Proudly, you declare the length of the room to be 5.356 meters (m) in length!

How certain are you that 5.356 m is the true length of the room? You recall that the uncertainty for any measurement is half the smallest division, so +/- 0.5 mm, or 0.0005 m. That seems wildly accurate and precise for a single measurement, and you suspect that your own measurement method prevented such high precision. Thinking on your method of measurement, first you begin to wonder how good of a job you did when you moved the meter stick from one position to the next. Did you truly put it exactly where it left off? Did that amount vary each time you did it? You certainly weren’t perfect, so how much were you off by? A few millimeters? Over the multiple measurements that could easily add up to being off by a centimeter or more. You also are unsure if it was always extra length added, or did you underestimate sometimes? Worried, you repeat the task again and this time you get 5 full meters, but the extra measurement this time comes out to be 34.3 cm given a total length of 5.343 m. A different value by 5.356 m – 5.343 m = 0.013 m! Clearly, you can’t be certain about the length of the room from these measurements alone. A fundamental rule of measurement dawns on you, ... All measurement has some level of uncertainty in it. A scientist’s goal is to reduce the uncertainty as much as possible. Entire theories may depend on it!

A few days later, you are still curious about how long your room really is. You ask a friend to come over and measure the room. You have shared your concern with not being able to perfectly move the meter stick and how you measured that last remaining bit and aren’t sure if that was the best method. Your friend decides that the best way to avoid this is to lay out a string from one end of the room. She then measures the length of the string, and she is very careful to mark each meter with marker before moving the meter stick. Using this method your friend determines the room to be 5.352 m long. You double check and measure the length of the string as well and get 5.348 meters long. This time only a difference of 5.352 m – 5.348 m = 0.004 m, or 4 mm!

You are beginning to feel more confident in the length of the room, but your friend has a few concerns. She noticed that there is a bit of extra material at the start and end of the meter stick, so the meter stick is likely slightly longer than a full meter. Being an acute observer, she noticed that the meter stick is a bit worn on the ends, and that she measured along the
edge while you measured in the middle of the meter stick. Even worse, she points out that if the millimeters were subdivided even more into $1/10,000^{th}$ of a meter, then you could have a more accurate answer still. Being a skeptic, she also brought up doubts about each demarcation being perfect. Perhaps one of the millimeter markers is really 1.1 mm and another is 0.96 mm. In frustration with the problem, you decide to take the midpoint of your two estimates and declare the room to be 5.350 m give or take a few millimeters. You have embraced the uncertainty and given your answer with some give or take a bit. The give or take a bit is a quantification of the uncertainty, or error, of a measurement. This is just a ballpark guess at the uncertainty. It would be great if you could truly quantify it.

1.4 Repeated Trials and Averages to Improve Measurements and Quantify Uncertainty

Your friend is not satisfied and wants to push the measurement further and to higher levels of accuracy. She is certain she can determine the true length of the room. She calls up her science major friends to come help. One of them has a tool with marks down to the $1/10,000^{th}$ (0.0001) meter. As a team, you all set to the task. You measure the following lengths in meters:

<table>
<thead>
<tr>
<th>Try</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>5.3488</td>
</tr>
<tr>
<td>2nd</td>
<td>5.3507</td>
</tr>
<tr>
<td>3rd</td>
<td>5.3512</td>
</tr>
<tr>
<td>4th</td>
<td>5.3391</td>
</tr>
<tr>
<td>5th</td>
<td>5.3510</td>
</tr>
<tr>
<td>6th</td>
<td>5.3517</td>
</tr>
<tr>
<td>7th</td>
<td>5.3602</td>
</tr>
<tr>
<td>8th</td>
<td>5.3506</td>
</tr>
</tbody>
</table>

One of the people just learned about the law of large numbers in his mathematics class that states that average of repeated trials (done with the same level of precision using the same method) will tend toward the true value as the number of trials increases. That is, the more trials (i.e., number of measurements made) the closer the averaged value will be to the “real” value. Averaging the eight trials together, and using a bit of statistics, you all find the average 5.3504 m with a standard deviation (standard error) of 0.0057 m. You decide this is good enough for your purposes and report the length of your room to be $5.3504 \pm 0.0057$ m, where ± means plus and minus and is the standard way of listing scientific uncertainty.

The above example is meant to point out that measuring something can be quite difficult and comes with many considerations. You could improve on the measurement through a better method (your friend with the string), and/or a more precise measuring instrument (a more finely divided measuring instrument). You can also make use of the law of large numbers and average many trials together to get a better answer. Regardless of all these efforts though, you could always add another decimal place to your answer, do more trials, and list smaller and smaller uncertainty. You could accurately measure the room to be $5.3505028 \pm 1.0 \times 10^{-7}$ m, but someone else could come along with a better measuring instrument and push that to 5.35050284 $\pm 1.0 \times 10^{-8}$ m, and so on and so forth. In theory could keep the one more decimal point game up until you get to the Planck length, which is theoretically the smallest physical distance measurable, but you still would have some problems. That number has its own uncertainty in it. It is calculated from three other fundamental physical constants of the universe: the speed of light in a vacuum, the Planck constant, and the gravitational constant. Two of these constants (the Planck constant and the gravitational constant) are empirically measured, themselves, and therefore have their own uncertainty in them! A major goal of astrophysics is to know such quantities as Planck’s constant and the gravitational constant to higher and higher accuracy, but such determinations depend on making other, almost always, indirect, measurements, which have their own limitations and uncertainty.
2. Collecting the Data

I hope you have gained an appreciation for how difficult and important measurement is to science. It requires incredible care and often relies on clever indirect measurement techniques. Today in lab, you will be engaging with some of those concepts, but in a very simplified, and hopefully fun, way that demonstrates core concepts of measurement methodology, accuracy, precision, and the how uncertainty is quantified. Measurements will be made, and the data for the whole class will be collected and examined to reinforce the concepts learned in this lab. **For this lab, you will work in student pairs.**

**Lab Activity 1 – Playing the Game and Taking Measurements**

For our exploration this lab’s core concepts, you will collect x- and y-position data of bean bags you will toss onto a board. You will also measure each bag’s angular location on the board with respect to a reference line. To increase interest, and hopefully have fun while doing so, your tossed bean bag locations on the board will be made into a game. The goal of the game is to get the highest score possible while also carefully measuring the (x, y)- positions. The general layout of the board and the rules of the game are shown in Figure 3.

**How to Play**
- Each member of a team throws their 5 bags onto the board. While one member throws, the other takes the measurements.
- Majority of the bag must be within the boundaries.

**Within Team Competition**
- The team member with the highest score wins.

**Between Team Competition**
- Total points of both players is the total team score. All teams are competing against one another for the highest score from the 10 throws.

**How to Measure**
- Each player measures the following:
  - The x- and y-position.
  - Use the grid drawn on the board to get x- and y-positions.
  - Measure to middle of the bag.
  - Do your best to estimate fractional distances between grid points.
  - The center of the board is (0, 0).

**Figure 3. The rules of the game. Two competitions will be taking place while you play. You will directly compete against your lab partner, and as a team, you and your lab partner will be competing with all other lab pairs. The goal is to get the highest total score. While one person throws the bean bags, their lab partner will record the x- and y-positions. The right-hand side of the board is positive x and radius. The top of the board is positive y.**
The Rules of the Game

Each lab partnership will have a turn at the game. You may want to take a couple practice tosses before starting. One person of the pair will throw their five bean bags while the other makes and records the following measurements: x-position, y-position, and of course, the score of each toss. The partners will then switch locations and repeat the process. At the end of each lab partnerships turn at the game, the pair should have 10 tosses with all the measurements recorded.

There are two competitions going on during this game. One is between partners. Highest score wins. The second is your combined team score versus all the other teams. Again, highest score wins. A bullseye is worth 100 points, and then from inner-most to outer-most rings, the scores for a bag in that ring are: 50, 30, and 15 points. Bags on the board, but not in a ring, are worth 5 points. Bags off the board are worth 0 points, and you have the additional challenge of still trying to get accurate measurements for the (x, y) position.

In cases when a bag lands on a boundary, the score is determined by the ring that containing the majority of the bag. This is roughly equivalent to the what ring the center of the bag is in.

Measurement Data Collection

After all the matches have been played, and every team has a score, you will need to enter your data into our online system using the provided Google Form – “Measurement Lab Data Entry Form.” Every team will have to enter their measurements into the form 10 times: one for each toss. In addition to the (x, y)-position, you will also have to calculate the radial distance away from the center. The radius, r, is calculated using the equation

$$ r = \sqrt{x^2 + y^2} $$

(3)

X- and y-values range from -5 to 5 using the grid on the board. Do your best to estimate fractional distances. The right-hand side of the board will be assigned positive radius values, and the left-hand side will be assigned negative radius values. An example of the measurements to be taken for each bag is shown in Figure 4.

![Figure 4](image)

**Figure 4.** An example of the data to be recorded in the Google Form. (x,y)-positions are determined using the grid on the board. Do your best to estimate fractional distances. The angle is measured using the provided protractors. The radius is calculated using Equation 3. The score is determined by the bags location on the board. Positive r values are the right-hand side of the board.
Remote measurements not related to the game

To gain experience with remote measurements and creative problem solving, the class will also perform remote measurements to determine the height of and distance to an object. If you are playing outside, the object will be the light pole across the street from the gameboard. If inside due to weather, the object will be the fire detector on the ceiling of the lab room. In both cases, you are allowed to measure the distance to or the height of the object directly. You must use triangulation methods, as depicted in Figure 5.

For this task, you will not be given any method on how to measure the distance to and height of the object. Your lab instructor will lead a class discussion where it is up to all of you to come up with a method to determine the distance and height via indirect measurements. Once your class has its method, representatives will take the necessary measurements and report their observations to the class.

\[
\tan a = \frac{H}{B} \\
H = B \tan a
\]

**Figure 5a.** Diagram of how altitude can be used to determine the height, \(H\), of an object given a known baseline distance, \(B\), and measuring the altitude angle of the target object, \(a\). Note that you will first have to determine the baseline distance to the board.

\[
\tan \theta = \frac{D}{B} \\
D = B \tan \theta
\]

**Figure 5b.** Diagram demonstrating how the distance triangulation technique can be used to determine the distance, \(D\), to a far-off object by measuring a baseline distance, \(B\), and measuring the angle between the baseline and the target object, \(\theta\)

Figure 5 demonstrates how to use triangulation to calculate distance and height. Figure 5a demonstrates that the height of an object can be calculated if you know the distance and can measure that object’s altitude angle \(a\). With known distance \(D\) and altitude angle \(a\), the height \(H\) can be determined as

\[
H = D \tan a 
\] (4)

Figure 5b demonstrates that the distance to an object can be calculated if you can measure a baseline distance \(B\) and an angle \(\theta\) measured from the far end of the baseline to the object with unknown distance \(D\). With known baseline length \(B\) and measured angle \(\theta\), the distance \(D\) to the object can be determined as

\[
D = B \tan \theta 
\] (5)
Lab Activity 2 – Examining the Distributions of Measurements

With all the data collected, it is time to calculate the radial distance away from the center of the board. Use Equation 3 \( r = \sqrt{x^2 + y^2} \) to determine \( r \). Once you have calculated all the radial distances, access the Google Form “Measurement Lab Data Entry Form” and enter your measurements. The form will prompt you to input the x-position, the y-position, the radial distance, and the score earned. Once finished with an entry, choose the prompt to do another entry. You will do this repeatedly until all the data on all 10 tosses is entered.

Once all the data is entered, your instructor will graphically display the results on the projector. The distributions for the x, y, angle, radius, and score will be displayed as histograms, which shows how frequently each measurement was made within a certain range. Using this graphical information, you will need to answer questions relating to what the distributions look like, the accuracy, and the precision of each distribution.

You will also calculate the averages of your bag trials and compare that to the overall class distribution.
# Measurement & Data Lab Student Worksheet

Name:  
Lab Instructor:  
Lab Meeting Time:  

## Measurement Data Records

Input your measured data in the table below. Put the name of each player under Player 1 and Player 2. Below the all the data entry, calculate your average x, y, radius, and score for all ten of your team’s measurements. If you need to save calculating the averages for the take-home portion of this lab, then that is okay. You can skip them for now if you do not have time.

### Measurement Data Table / Game Scorecard

<table>
<thead>
<tr>
<th>Toss #</th>
<th>x</th>
<th>y</th>
<th>Angle</th>
<th>Radius</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Player 1:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total Score

| **Player 2:** |   |   |       |        |       |
| 1      |   |   |       |        |       |
| 2      |   |   |       |        |       |
| 3      |   |   |       |        |       |
| 4      |   |   |       |        |       |
| 5      |   |   |       |        |       |

Total Score

Player 1 + Player 2 Scores = Team Score

<table>
<thead>
<tr>
<th>Team Averages</th>
<th>x average</th>
<th>y average</th>
<th>Average angle</th>
<th>Average radius</th>
<th>Average score</th>
</tr>
</thead>
</table>


**Indirect Measurement Results**

Record the measurements you made to determine the official game play distance and height for the bag tossing platform. Use these measurements to calculate the distance and height.

1. Describe the method the class used to determine the distance to and height of the light pole or fire detector (if lab took place inside).
   a. [Word description of method] Describe the complete method the class used including all angles and distances measured.
   b. [Graphical description of method] Draw one or more diagrams demonstrating the method you used. Label all distances and angles measured or determined.

2. What is the distance to and height of the object?

   Height in meters: ________________
   Distance in meters: ________________

3. Describe at least 3 sources of error that would have propagated into the final answer. That is where could better measurements have been made?
4. Describe at least 2 ways you would improve the accuracy and precision of the distance and height if you had more time than this lab allowed. Assume you really want to know the distance and height.
Questions Relating to Distributions of the Measurements

5. Using the provided axes below, sketch the overall distributions of all the measurements from today’s lab. Your instructor should have these distributions displayed on the projector.

**Sketches of Measurement Distributions**
*for entire class’s data*

<table>
<thead>
<tr>
<th>Distribution of x-positions</th>
<th>Distribution of y-positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average:</td>
<td>Average:</td>
</tr>
<tr>
<td>Standard Deviation:</td>
<td>Standard Deviation:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Currently Not Included</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
</tr>
<tr>
<td>360°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distribution of angles</th>
<th>Distribution of Radii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average:</td>
<td>Average:</td>
</tr>
<tr>
<td>Standard Deviation:</td>
<td>Standard Deviation:</td>
</tr>
</tbody>
</table>

6. Do all the distributions have roughly the same shape? If not, which one(s) is(are) the different, and how are they different?
7. For the x-position, y-position, and radius, we should expect the average distance to be zero (a bull’s eye).
   a. Are the class’s distributions of these measurements near the expected, “true” value?
   b. If an average is not close to the expected value of 0 then the data is telling us something about an error inherent to the measurement. This is what we call a **systematic error**, which an error from the method used to obtain the data. Such an error can negatively affect our ability to appropriately test hypotheses in science. Do you observe any systematic errors in the data? If yes, what do you think may have caused them?

8. The width of the distributions characterizes the precision of the measurement. A narrow distribution would mean that every toss was close to the average with little variance. This would be high precision. A wide distribution means that the position measurements varied widely from toss to toss. This would be low precision.
   a. Do the x-position and y-position distributions have the same precision? If not, which is more precise?
   b. 
   c. What could have caused any differences in the x-position and y-distribution precisions?

Note: the width of a distribution, or how much each toss varies from other tosses is capture by the quantity called the standard deviation, which is one of the displayed values. The standard deviation gives the uncertainty.
9. The radial distance distribution combines information from the x-positions and the y-positions. Therefore, characteristics of the x-position distribution and y-position distribution should be present in the radial distance distribution.
   a. Describe the shape of the radial distribution with respect to the shape of the x- and y-position distributions.
   b. Are there any systematic offsets present in the radial distances? If so, provide your hypothesis on what caused the offset.
   c. Describe how you could use bean bags and the target board to test your hypothesis. What experiment would you run to either confirm there is no systematic offset, or to confirm your explanation of what caused the offset.
10. Any measurement has some inherent error in it. You certainly had some when measuring the x- and y-positions of the bean bags. List and describe at least **TWO** sources of error you encountered when measuring a bag’s x- and y-positions. That is, what things that you can think of made it difficult to measure the bags exact (x, y)-location?

**Lab Follow-up Questions**

[Complete the following questions at home if you do not have time to during class.]

11. In your own words, write definitions for **accuracy**, **precision**, and **uncertainty**.

12. Use the triangulation method to determine the distance d using the image to the right. The baseline distance is 5 meters, and the measured angle is 70°.

13. Describe the difference between a direct and an indirect measurement.
14. **Uncertainty in Distance from Uncertainty in Parallax**

Consider a far-off star in our galaxy. Using the method of stellar parallax described in Section 1.2.3, you measure the star’s parallax to be $0.002 \pm 0.001$ arc-seconds. You feel you have done a great job to get that level of precision in your measurement. Let’s see how that small uncertainty of 0.001 arc-seconds translates to uncertainty in distance.

a. Use the measured value of 0.002 to calculate the distance to the star in parsecs and convert that to light-years (1 parsec = 3.26 light years).

\[
\text{Distance} [\text{pc}] = \frac{1}{p[\text{arcsec}]}
\]

Distance in parsecs: ________________ pc
Distance in light-years: ________________ ly

b. Use the measured value minus the uncertainty $0.002 - 0.001 = 0.001$ pc to calculate the distance to the star in parsecs and convert that to light-years (1 parsec = 3.26 light years).

\[
\text{Distance} [\text{pc}] = \frac{1}{p[\text{arcsec}]}
\]

Distance in parsecs: ________________ pc
Distance in light-years: ________________ ly

c. Use the measured value plus the uncertainty of $0.002 + 0.001 = 0.003$ pc to calculate the distance to the star in parsecs and convert that to light-years (1 parsec = 3.26 light years).

\[
\text{Distance} [\text{pc}] = \frac{1}{p[\text{arcsec}]}
\]

Distance in parsecs: ________________ pc
Distance in light-years: ________________ ly

d. Based on your calculations, comment on how well you have determined the distance to the star.