Scientific Measurement

How to determine the unknown

Author: Sean S. Lindsay
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Learning Goals

In this lab, students will engage with measurement, estimation, problem solving, and error analysis. At the completion of this lab, students will

- Perform simple scientific measurements
- Learn how to calculate surface area and volume
- Understand why units are important to science and effective communication
- Learn how to measure angles using a protractor
- Learn how angular measurements can be used to determine values such as height and distance using simple geometry
- Explore uncertainty and error inherent to all scientific measurement
- Engage with problem solving in the face of the unknown

Materials

- String, meter stick, protractor, clinometer, painter’s tape, calculator
- Five Prepared Stations: Distance Station with marked tracks; Surface Area Station, Volume Station with 2 boxes, marble, large sphere, and jar; Angle Measurement Station, and Altitude-Triangulation Station
- Prepared Google response forms

Pre-lab Questions

1. In your own words, write definitions for accuracy, precision, and uncertainty.
2. Measure the length between your ankle and your knee in number of “finger widths,” where a finger width is roughly the width of one of your fingers (not thumb).
3. Use the triangulation method to determine the distance d using the image to the right. The baseline distance is 5 meters, and the measured angle is 70°.
4. Describe the difference between a direct and an indirect measurement.

1. Background

1.1 Introduction to Scientific Measurement, Estimation, and the Unknown

How do we go about determining things like the number of galaxies in the universe, how much “stuff” the universe contains, the value of the gravitational constant of the universe, and the distance to the stars or how bright they are? Where do we get the evidence to evaluate some of our biggest astronomical questions, such as how did the universe, the
solar system, and Earth get their start? To answer all of these questions, scientists must collect observations and empirical evidence to test the predictions of their hypotheses and theories against. The collection of these observations and evidence almost certainly involves measuring some quantity of something. Whether that be the amount of light being received by a distant supernova, the total amount of mass (both luminous and dark) in a galaxy cluster, or the distance to the stars of the Milky Way, observations must be made, and data must be collected. How well we can measure the related quantities is intimately tied to how well we can evaluate our deepest scientific understandings of the universe.

Answering some of the above questions is a daunting task, and often times we only have a ballpark idea of what the answer might be. In more intriguing cases, we find ourselves in completely new scientific territory where we have little theory to rely upon. Here, science must tackle challenging questions in face of the unknown. We may have no known methods from which we can start trying to solve the problem. Often times, there are multiple ways to address a question, some of which will provide more reliable answers than others. We are left to rely on our own creativity and ingenuity to develop new problem-solving techniques. With hypothesis, theory, and ingenuity as our resources, we then have to determine, using the principles of science, which methods are best. This frequently comes down to how those methods make their measurements, what they are measuring, and which one gives the explanations we have the most confidence in.

In this lab, we will explore the some of the techniques scientists and astronomers use to begin addressing how to approach scientific problems and gather the required evidence. We will explore the difficulty in making measurements, the uncertainty and errors associated with measurement, and how we can compare measurements of the same thing made with different tools or techniques. We will also explore the process of scientific estimation, which relies on empirical measurements and the laws of mathematics. A final, and more general, goal of this lab is the subtle art of problem solving in the face of the unknown.

1.2 Scientific Measurement

Often, measurement is thought of as a simple task. For most people, daily life measurements are simply measuring how long something is with a measuring stick, how much volume you need with a measuring cup, the mass of something with a scale or a mass balance, or how long something takes with a stopwatch. If you are off by a few centimeters, milliliters, grams, or seconds, then so be it, there was no real harm done. In science, however, we strive to arrive at the real value as close as possible, or rather, to have measurements with high levels of accuracy. To have an accurate measurement, or set of measurements, is to be close to the real value with the measurement, or the average of the set of measurements. To accomplish accurate measurements, much more care and sophisticated instruments are needed than you experience in everyday life.

In science, we also never rely upon a single measurement. Science demands that we make measurements repeatedly to check and re-check our evidence. In these repeated trials, scientist aim for measurement methods that consistently give you values that are nearly the same, or rather, have high levels of precision. How close they are together, or how precise your measurements are, determines how certain we can be that our evidence provides a test for the predictions of our hypotheses and theories. In other words, the precision of our measurements quantifies the uncertainty in our measurements, or rather, what is the probability that our measurement is accurate within a certain range of values. Altogether,
scientists strive to measure as close to the true value as possible (accuracy) and repeatedly get measurements that are near to the same value (precision), which in turn, allows them to understand, with statistical confidence, they have homed in on the true value plus or minus a little bit (uncertainty). It should be noted that a real, or true, measured value\(^1\) is an ideal concept. It exists, but it is impossible to measure exactly.

1.2.1 Direct versus Indirect Measurement

The examples of measuring distance, volume, mass, and time given above are all examples of direct measurement. This is the simplest and most ideal case of the quality of your measurement coming down to the precision of your measuring instrument. For example, if you have a meter stick that marks centimeters and millimeters, you can measure down to the millimeter plus or minus a half millimeter. The plus or minus half a millimeter (+/- 0.5 mm) is the uncertainty of the measuring instrument. A general rule of thumb in measurement is that the uncertainty of an instrument is half the smallest division on the instrument. Another example is measuring the mass of something on a digital scale. If the digital scale gives the mass in grams to one decimal place, then the uncertainty is +/- 0.05 grams.

In many scientific fields, and especially in astronomy, scientists do not have the luxury of making direct measurements. Instead, they must rely upon indirect measurement, which involve techniques that exploit a quantity's dependency (often mathematical) on something that can be directly measured. For example, astronomical objects (e.g., the Moon, planets, stars, galaxies, etc.) are physically too far away to measure directly, so astronomers must rely on indirect techniques to measure their distances. Measuring distances accurately and precisely turns out to be perhaps the most difficult task astronomer’s face! A few examples that you will explore in the lab activities and lab questions of this are the techniques to determine the distances to stars and planets.

1.2.2 Distance from radar methods

The distances to planets (and the Moon) are made using radar (Radiation Detection And Ranging). In this indirect distance measurement, astronomers time how long it takes the radio wave (a type of light) to travel to the object, reflect off it, and travel back to Earth. The radio wave travels at the speed of light (3.0 x 10\(^8\) meters per second), so how long the journey takes can be mathematically translated into a distance via

\[
\text{Distance} = \text{velocity} \times \text{time} \tag{1}
\]

Where the velocity is the speed of light, c=3.0x10\(^8\) m/s and the time is the measurement made. In indirect measurements the accuracy, precision, and uncertainty of the desired quantity depends on the accuracy, precision, and uncertainty of what is measured. In the case above, the uncertainty in the time measurement must be translated through to get the uncertainty in the distance. This translation of uncertainty from the direct measurement to the indirect one is called propagation of uncertainty (sometimes propagation of error).

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\(^1\) As opposed to a counted value, which has a distinct number. In the case of counting very large numbers, however, it may be intractable to determine a true countable number.
1.2.3 Distance from stellar parallax

In the other example, to measure the distances to stars, astronomers use the technique of stellar parallax, which is a distance triangulation technique. **Triangulation** is a method to determine the distances between two points through drawing series of (right) triangles, where the distance of one side of the triangle can be directly measured, and at least one of the two non-right angles can be measured. With a known baseline distance, and a measured angle of a right triangle, it is possible to calculate the distance of a far-off object.

Astronomers use a similar technique, but the angle measured is determined by the phenomenon known as parallax, which is the angular displacement an object appears to have against a fixed background when viewed from two different positions. Referred to as stellar parallax, the general method is shown in Fig. 1. The baseline is the distance between the two vantage points. Astronomers use the diameter of Earth’s orbit as the baseline distance because it is the largest baseline accessible to humans. The angular displacement is the apparent angular shift a star appears to have when viewed from opposite sides of Earth’s orbit. Here the direct measure is the displacement angle. Half of that measured displacement is called the parallax angle, $p$, which often is just referred as the stellar parallax [See Fig. 1]. It can be translated into distance using

$$
Distance \ [pc] = \frac{1}{p \text{[arcsec]}}
$$

(2)

Since the stars are very far away, the stellar parallax angle is very small and hard to measure. This makes it prone to high levels of uncertainty, which propagate to a large uncertainty in distance. In astronomy, measuring distances is extraordinarily difficult!

![Diagram of stellar parallax](image)

*Figure 1. The method of stellar parallax to determine the distances to nearby stars.*

1.3 The Difficulty of a Scientific Measurement – A Direct Measurement Example

Scientific measurement is a deceptively difficult task with a large amount of nuance. Consider the simple task of measuring exactly how long a room is. For simplicity, let us imagine that the room’s floor print is a square, so that the length and width are equal. How would you go about measuring the length of the room? Likely, you would get a measuring tape, a meter/yard stick, or some other standard of length measurement and just see how
long the room is. Since we are working in the world of science, let us imagine that you chose to measure the length in meters. Unfortunately, you only have a meter stick in your room, but nevertheless, it seems like a good one with centimeters (cm) and millimeters (mm) clearly marked. So, you set to the task with confidence that you can determine the length of the room. You measure 5 full meters and have a bit leftover where the meter stick is too long to measure the remaining distance to the wall. Being clever, you mark the 5-meter mark on the floor and then measure from the wall out to that point to and find that length to be 35.6 centimeters. Proudly, you declare the length of the room to be 5.356 meters (m) in length!

How certain are you that 5.356 m is the true length of the room? You recall that the uncertainty for any measurement is half the smallest division, so +/- 0.5 mm, or 0.0005 m. That seems wildly accurate and precise for a single measurement, and you suspect that your own measurement method prevented such high precision. Thinking on your method of measurement, first you begin to wonder how good of a job you did when you moved the meter stick from one position to the next. Did you truly put it exactly where it left off? Did that amount vary each time you did it? You certainly weren’t perfect, so how much were you off by? A few millimeters? Over the multiple measurements that could easily add up to being off by a centimeter or more. You also are unsure if it was always extra length added, or did you underestimate sometimes? Worried, you repeat the task again and this time you get 5 full meters, but the extra measurement this time comes out to be 34.3 cm given a total length of 5.343 m. A different value by 5.356 m – 5.343 m = 0.013 m! Clearly, you can’t be certain about the length of the room from these measurements alone. A fundamental rule of measurement dawns on you, ... **All measurement has some level of uncertainty in it.** A scientist’s goal is to reduce the uncertainty as much as possible. Entire theories may depend on it!

A few days later, you are still curious about how long your room **really** is. You ask a friend to come over and measure the room. You have shared your concern with not being able to perfectly move the meter stick and how you measured that last remaining bit and aren’t sure if that was the best method. Your friend decides that the best way to avoid this is to lay out a string from one end of the room. She then measures the length of the string, and she is very careful to mark each meter with marker before moving the meter stick. Using this method your friend determines the room to be 5.352 m long. You double check and measure the length of the string as well and get 5.348 meters long. This time only a difference of 5.352 m – 5.348 m = 0.004 m, or 4 mm!

You are beginning to feel more confident in the length of the room, but your friend has a few concerns. She noticed that there is a bit of extra material at the start and end of the meter stick, so the meter stick is likely slightly longer than a full meter. Being an acute observer, she noticed that the meter stick is a bit worn on the ends, and that she measured along the edge while you measured in the middle of the meter stick. Even worse, she points out that if the millimeters were subdivided even more into 1/10,000th of a meter, then you could have a more accurate answer still. Being a skeptic, she also brought up doubts about each demarcation being perfect. Perhaps one of the millimeter markers is really 1.1 mm and another is 0.96 mm. In frustration with the problem, you decide to take the midpoint of your two estimates and declare the room to be 5.350 m give or take a few millimeters. You have embraced the uncertainty and given your answer with some give or take a bit. The give or
take a bit is a quantification of the uncertainty, or error, of a measurement. This is just a ballpark guess at the uncertainty. It would be great if you could truly quantify it.

1.4 Repeated Trials and Averages to Improve Measurements and Quantify Uncertainty

Your friend is not satisfied and wants to push the measurement further and to higher levels of accuracy. She is certain she can determine the true length of the room. She calls up her science major friends to come help. One of them has a tool with marks down to the 1/10,000th (0.0001) meter. As a team, you all set to the task. You measure the following lengths in meters:

<table>
<thead>
<tr>
<th>1st Try</th>
<th>2nd Try</th>
<th>3rd Try</th>
<th>4th Try</th>
<th>5th Try</th>
<th>6th Try</th>
<th>7th Try</th>
<th>8th Try</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3488</td>
<td>5.3507</td>
<td>5.3512</td>
<td>5.3391</td>
<td>5.3510</td>
<td>5.3517</td>
<td>5.3602</td>
<td>5.3506</td>
</tr>
</tbody>
</table>

One of the people just learned about the law of large numbers in his mathematics class that states that average of repeated trials (done with the same level of precision using the same method) will tend toward the true value as the number of trials increases. That is, the more trials (i.e., number of measurements made) the closer the averaged value will be to the “real” value. Averaging the eight trials together, and using a bit of statistics, you all find the average 5.3504 m with a standard deviation (standard error) of 0.0057 m. You decide this is good enough for your purposes and report the length of your room to be 5.3504 ± 0.0057 m, where ± means plus and minus and is the standard way of listing scientific uncertainty.

The above example is meant to point out that measuring something can be quite difficult and comes with many considerations. You could improve on the measurement through a better method (your friend with the string), and/or a more precise measuring instrument (a more finely divided measuring instrument). You can also make use of the law of large numbers and average many trials together to get a better answer. Regardless of all these efforts though, you could always add another decimal place to your answer, do more trials, and list smaller and smaller uncertainty. You could accurately measure the room to be 5.3505028 ± 1.0x10⁻⁷ m, but someone else could come along with a better measuring instrument and push that to 5.35050284 ± 1.0x10⁻⁸ m, and so on and so forth. In theory could keep the one more decimal point game up until you get to the Planck length, which is theoretically the smallest physical distance measurable, but you still would have some problems. That number has its own uncertainty in it. It is calculated from three other fundamental physical constants of the universe: the speed of light in a vacuum, the Planck constant, and the gravitational constant. Two of these constants (the Planck constant and the gravitational constant) are empirically measured, themselves, and therefore have their own uncertainty in them! A major goal of astrophysics is to know such quantities as Planck’s constant and the gravitational constant to higher and higher accuracy, but such determinations depend on making other, almost always, indirect, measurements, which have their own limitations and uncertainty.

1.5 Scientific Estimation

There is still much about the universe that remains unknown, and it is the job of the astronomer to attempt to turn that unknown into the known. Consider the case of an unknown where there is an answer, but there is no practical means to measure it. The example of how many galaxies exist in the observable universe (the part we can actually observe, the universe is certainly larger) is a perfect example of this. We simply do not have
the time, technology, or data to detect every single one and count them up. We know that there must be an answer, a “true” number of current galaxies in part of the universe we can see. Addressing this question is a prime example of a scientific estimation, which provides a meaningful estimate based on scientific methods and principals. As part of the take-home portion of this lab, you will estimate the number of galaxies in the observable universe using one of the deepest images the Hubble Space Telescope has ever taken. This image is called the Hubble Ultra-Deep Field (HUDF) and in a 2.4’ x 2.4’ (‘ is arc-minute) image, Hubble observed over 10,000 galaxies!

A good scientific estimation hinges upon scientific and mathematical understanding (theory), a solid starting point that is often a smaller version of the problem, and then extrapolating the measurements you make in the smaller version to the whole. Notice that part of a good estimation always will require observations and measurements, which will have some uncertainty in them. For the final estimation answer, these uncertainties will have to be propagated through the estimation process to arrive at an estimate with listed uncertainties indicating how confident you are in that estimate.

2. Measuring Exercises

I hope you have gained an appreciation for how difficult and important measurement is to science. It requires incredible care and often relies on clever indirect measurement techniques. Today in lab, you will be engaging with some of those concepts, but in a very simplified way that demonstrates core concepts of measurement methodology, quality of measurement, scientific estimation, and problem solving in the face of not knowing the right answer. You will not be given any “real” values for what you are trying to measure, and you will rely on knowledge of accuracy, precision, uncertainty, and the law of large numbers to evaluate the quality of your measurements. Good luck and have fun!

For this lab, you will work in student pairs.

Lab Activity 1 – Measuring Stations

Your lab instructor will give you a length of string. This will be your tool to measure physical distances, and we will define the unit of “strings” for this lab. It is a crude instrument, and you are encouraged to get creative in the ways you use it (how can you subdivide it for precise smaller measurement divisions?). You are NOT ALLOWED TO WRITE ON OR CUT THE STRING. YOU ARE ALSO NOT ALLOWED TO WRITE ON OR ALTER ANY OF THE OBJECTS BEING MEASURED.

Pre-lab 1: Guess the number of marbles in the jar. Write your guess in the space provided in Table 3.1 on the “Student Worksheet.” This is one of the values you will have to report to the Google Form before leaving today.

Pre-lab 2: On the Student Worksheet, please trace out the length of your string along the right margin.

Station A – Measuring Distance

Last lab period, you measured the distance marked on the floor using your strings. Today’s activity is an exercise in increasing precision. Using one of the provided meter sticks,
measure the length of the distance you measured last week to the nearest millimeter. Take note of how you perform the measurement and report your general method and how you did fractional strings (lengths shorter than your string) on the “Student Worksheet.” Report your measured distance to Table 3.1 and input it on the Google Form.

**Station B – Measuring Surface Area**

Use your string for these measurements. You task at this station is to measure the dimensions (e.g., length, width, circumference, diameter, etc.) of printed shapes located at the Surface Area station. The shapes are a rectangle, a right triangle, and a circle. The measurement of the circle requires you to measure the diameter of the circle and the circumference. For this first lab activity, simply measure these dimensions and report them on Table 3.1.

*The Google Form Response item for this station will be the calculation your make for the area of the right triangle.*

**Station C – Measuring Volume**

You task at this station is to measure the dimensions (e.g., length, width, circumference, diameter, etc.) of the three-dimensional objects located at the Volume Station. These objects are a large sphere, a marble, and a cylindrical jar. The jar is identical to the jar containing marbles, and the marble is identical to the marbles in the jar.

*The Google Form Response item for this station will be the calculation your make for the volume of the large sphere.*

**Station D – Measuring Angles**

This station is designed to get you used to measuring angles using a protractor. Notice that the protractor is a much more precise instrument than the lengths of string you have been using. This means you should be able to get much more accurate measurements with high precision. This will become apparent when you look at the class distribution of responses for this station compared with the “string” measuring stations.

Your task at this station is to use a protractor to measure angles with as much accuracy as possible. You will measure the angle between two lines, an indicated angle of a right triangle, and the number of degrees around indicated arcs of a circle. The right triangle in this station is the same as the right triangle in Station B – Measuring Surface Area.

*The Google Form Response item for this station is protractor measured angle of the right triangle AND a trigonometric calculation of that angle using the lengths of the sides of the triangle made in Station B.*

**Station E – Indirect Measurements – Triangulation and Altitude**

In this station, you will engage with indirect measurement techniques of determining distance from angular measurements. This is a very simplified version of the method of parallax that astronomers use to measure the distances to stars. The goal is to get an appreciation for how difficult it is to accurately determine the distances to objects from indirect measurement techniques. The trick is to have accuracy and precision in your indirect measurement since uncertainty increases through the use of equations to translate to the desired measurement, which is referred to as the propagation of uncertainty. Small
uncertainty in the indirect measurement might lead to large uncertainty in the quantity you are interested in. Hence, it is of utmost importance to have high precision measurements and to use clever indirect measurements that minimize the increase of the propagated uncertainty.

Your tasks are to perform two altitude measurements and one distance triangulation measurement using known baseline distances. A diagram of how to determine height from altitude measurements and distances from triangulation measurements are shown in Fig. 2.

For the altitude measurements, you will use a crude clinometer (device to measure altitude angle) to measure the altitude angle for the projector and for the fire detector on the ceiling. You will make these measurements from the X taped to the floor, which have the baseline distances of 6.7 meters between X and the projector and 6.8 m between X and the fire detector. **Have both members of your group measure the angle.** Record your altitude angles and their averages on Table 3.1. You also need to provide a sketch of the distances, angles, and any other relevant factors the demonstrates the altitude measurements.

For the distance triangulation measurement, you will use a crude angular separation instrument (the protractor with attached guide arrows) to measure the non-right angle between the baseline and the target object, which is labeled as the “desired angle” $\theta$ in Fig. 2b. Your goal is to remotely measure the length of the hallway outside the lab room from the lab room to the double doors. The piece of painter’s tape on the hallway floor is 1 meter away from the left wall (facing the double doors at the end of the hallway). **Have both members of your group measure the angle.** Record the angles you measured and their average on Table 3.1. You also need to provide a sketch of the distances, angles, and any other relevant factors the demonstrates the triangulation measurements.
Student Worksheet

3. Lab Activity 1 - In-class Measurement Records

1. Trace out the length of your string in the right Margin
2. Complete Table 3.1 – Student Measurement Data Table

Table 3.1 – Student Measurements Data Table

<table>
<thead>
<tr>
<th>Pre-lab Station – How many marbles in the jar?</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Guess – Report to Google Form</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station A – Measuring Distance</th>
<th>Unit: millimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length of Distance - Report to Google Form</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station B – Measuring Surface Area</th>
<th>Unit: “Strings”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
<td></td>
</tr>
<tr>
<td>Length (long side)</td>
<td></td>
</tr>
<tr>
<td>Width</td>
<td></td>
</tr>
<tr>
<td><strong>Right Triangle</strong></td>
<td></td>
</tr>
<tr>
<td>Length (long side)</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td></td>
</tr>
<tr>
<td><strong>Circle</strong></td>
<td></td>
</tr>
<tr>
<td>Diameter of Circle</td>
<td></td>
</tr>
<tr>
<td>Circumference of Circle</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.1 continued– Student Measurements Data Table

<table>
<thead>
<tr>
<th>Station C – Measuring Volume</th>
<th>Unit: “Strings”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Sphere (report the one you measured)</td>
<td></td>
</tr>
<tr>
<td>Circumference</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td></td>
</tr>
<tr>
<td>Marble – (report the one you measured)</td>
<td></td>
</tr>
<tr>
<td>Circumference</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td></td>
</tr>
<tr>
<td>Other? [Describe and write value for either diameter or circumference]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jar</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td></td>
</tr>
<tr>
<td>Circular base (report the one measured)</td>
<td></td>
</tr>
<tr>
<td>Circumference</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station D - Angles</th>
<th>Unit: Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle Between Arrows</td>
<td></td>
</tr>
<tr>
<td>Right Triangle Angle $\theta$ – Report to Google</td>
<td></td>
</tr>
<tr>
<td>Degrees Around Circle from point 1 to 2</td>
<td></td>
</tr>
<tr>
<td>Degrees Around Circle from point 2 to 3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station E – Remote Measurement</th>
<th>Unit: Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hallway Length Triangulation Angle$^{6f}$</td>
<td>P1: P2: Avg:</td>
</tr>
<tr>
<td>Altitude of Projector</td>
<td>P1: P2: Avg:</td>
</tr>
<tr>
<td>Altitude of Fire Alarm</td>
<td>P1: P2: Avg:</td>
</tr>
</tbody>
</table>

*P1 and P2 are Person 1 and 2. Each group member takes a measurement.*
4. Lab Activity 2 – Questions and Calculations for Google Form

The second lab activity is to perform the set of calculations that you will submit to the Google Form, “Measurement Lab Response Form.” These are the required in-class calculations and are logged on Table 4.2 Student Calculations. The calculations required for Google Form responses are indicated by bold italics font with the GF-superscript (in blue in the color version). Be aware that you will have to complete all calculations for the take-home portion of this lab.

Table 4.1. Length, Area, and Volume Equations

<table>
<thead>
<tr>
<th>Length Equations</th>
<th>Area Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of circle</td>
<td>rectangles: $A = l \times w$</td>
</tr>
<tr>
<td>Circ. of circle</td>
<td>triangle: $A = \frac{1}{2} b \times h$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box: $V = l \times w \times h$</td>
</tr>
<tr>
<td>Cylinder: $V = \pi R^2 \times h$</td>
</tr>
<tr>
<td>Sphere: $V = \frac{4}{3} \pi R^3$</td>
</tr>
</tbody>
</table>

- $V$ is Volume
- $R$ is Radius
- $A$ is area
- $l$ is length
- $w$ is width
- $h$ is height

For equations related to length, area, and volume, please refer to Table 4.1. All other calculations will have equations provided for you in the questions.

Questions Related to Station A - Distance

3. Describe the general method you used to determine the distances for the marked distance

   a. What was your measured distance in millimeters (mm)? Report to Google

   b. Describe the method you used to get your final distance in mm. How confident are you in its accuracy and precision? If you are having trouble, please read Sections 1.3 and 1.5.
c. Your instructor will show you the distribution of all the lengths measured during last week’s lab. Describe the distributions using terminology used by your instructor and terms introduced in this lab?

What is the average distance over all students?
Blue Strings: White Strings:

<table>
<thead>
<tr>
<th>Blue String length (mm)</th>
<th>White String length (mm)</th>
</tr>
</thead>
</table>

d. Using your knowledge on how to convert units, and the conversion factor between “blue strings” and “white strings” you determined at the beginning of this lab, determine the length in mm for the blue string and the white string.

Questions Related to Station B – Surface Area (Refer to Table 4.1 for equations)

4. Calculate the area of the rectangle and right triangle in “square strings.” Report the right triangle area to your Google Form

| Rectangle Area = | “square strings” |
| Right Triangle Area = | “square strings” |

5. Circle Dimensions: For you circle measurements, determine the following:
a. Calculate the radius in “strings” of the circle using your measurement for its circumference or diameter.
Radius = “strings”

b. Calculate the radius in “strings” of the circle using your measurement for its diameter.
Radius = “strings”

c. Are your two radii calculations equal [Yes/No]? Which of the two radii values do you think has the least amount of measurement error in it, and what leads you to that choice?
d. What is the percent difference between your two radii? \( |value| \) is absolute value.

\[
Percent \ Difference = 100 \times \frac{|(First \ Value) - (Second \ Value)|}{(Average \ of \ the \ two)}
\]

Percent Difference = ____________%.

6. Circle Area: For you circle measurements, determine the following:
   a. The area of the circle in “square strings” using your radius from its diameter.
      Area = ___________________ “square strings”
   
   b. The area of the circle in “square strings” using your radius from its circumference.
      Area = ___________________ “square strings”
   
   c. What is the percent difference between your two circle areas?
      Percent Difference = _________________%.
   
   d. Errors compound when propagating them through equations. Between your radius and area calculations, which one has the larger percent difference? Comment on why you think that one has the larger error.

**Questions Related to Station C – Volume**

7. Answer the following questions about the volume of the jar.
   a. Calculate the volume of the jar in “cubic strings.”
      Volume = _________________ “cubic strings”
b. Describe the method you used to determine the dimensions and ultimately the volume of the jar. Did you assume it was a simple cylinder?

8. Answer the following questions about the volume of the large sphere.
   a. Calculate the surface area and \textit{volume of the large sphere in “cubic strings.”} 
      \textit{Report the volume vto your Google Form}
      
      \begin{align*}
      \text{Surface Area} &= \underline{\phantom{0000}} \text{ “square strings”} \\
      \text{Volume} &= \underline{\phantom{0000}} \text{ “cubic strings”}
      \end{align*}

b. Describe how you determined the radius of the large sphere?

9. Answer the following questions about the volume of the marble.
   a. Calculate the volume of the marble in “cubic strings.”
      \begin{align*}
      \text{Volume} &= \underline{\phantom{0000}} \text{ “cubic strings”}
      \end{align*}

\textbf{Questions Related to Station D – Angles with a protractor}

10. Answer the following questions about the right triangle.
    a. \textit{Calculate the value of the angle $\theta$ in the right triangle using trigonometry.} 
       \textit{Report this value and your protractor measurement to the Google Form}
       \begin{align*}
       \tan(\theta) &= \frac{\text{Opposite}}{\text{Adjacent}} \Rightarrow \theta = \arctan\left(\frac{\text{Opposite}}{\text{Adjacent}}\right) \\
       \text{Angle}, \theta &= \underline{\phantom{0000}} \text{ degrees}
       \end{align*}

b. What is the percent difference between your measured value for $\theta$ and your calculated via trigonometry value?
   \begin{align*}
   \text{Percent Difference} &= \underline{\phantom{0000}} \%
   \end{align*}
c. Comment on the two methods of determining the angle. Which method do you think gives a more reliable answer to the true value, the direct measurement, or the calculation based on indirect measurements?

11. Assume that “degrees around a circle” measurement is really a diagram of Earth orbiting the Sun. Over the course of one year (365.25 days), the Earth will revolve 360°.
   a. What fraction of one full year does your measurement between points 1 and 2 represent?

   b. How many days on Earth does your measurement around the circle represent?

**Questions Related to Station E – Triangulation and Altitude**

12. Answer the following questions about the angle you measured for determining distance of the hallway to the double doors remotely via triangulation.
   a. Draw the right triangle you used to determine the distance of the hallway to the double doors. Label the baseline with “baseline = 1 m”, the angle you measured, and the unknown distance to the doors.

   **Drawing of Distance Triangulation**

   b. *GF Determine the distance to the door using the average of the two angle measurements.*

   **Report this value and the angle you measured to your Google Form**

   \[
   \text{Distance} = \text{Baseline} \times \tan(\text{angle})
   \]

   Distance = _____________m. Angle = ________________ degrees
13. Answer the following questions about the angle you measured for determining the heights to the projector and to the fire detector.
   a. Draw a picture demonstrating the altitude to the projector and the fire detector. The baseline distance on the floor between and the blue tape X on the floor is 6.7 meters the projector and 6.7 meters and for the fire detector. Be sure to label all parts.

<table>
<thead>
<tr>
<th>Drawing of Projector Altitude</th>
<th>Drawing of Fire Detector Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Drawing of Projector Altitude" /></td>
<td><img src="image2.png" alt="Drawing of Fire Detector Altitude" /></td>
</tr>
</tbody>
</table>

b. **Determine the height to the projector using the average of the two angle measurements.** Do your best to get the true height above the floor. Describe how you got your final answer.

   *Report this value to your Google Form*
   
   \[ \text{Height} = \text{Baseline} \times \tan(\text{angle}) \]

   Height = ________________ m

c. **Determine the height to the fire alarm using the average of the two angle measurements.** Do your best to get the true height above the floor. Describe how you got your final answer.

   *Report this value to your Google Form*
   
   \[ \text{Height} = \text{Baseline} \times \tan(\text{angle}) \]

   Height = ________________ m.
14. Complete all the spaces in Table 4.2 Student Measurement Calculations that are required for submission on the Google Form, “Measurement Lab Response Form.” These values appear in *bold italics* and are indicated with the *GF*-superscript.

**Questions Related to Scientific Estimation & Uncertainty**

15. For the Station C - Volume Measurement, you measured the physical dimensions and calculated the volume of a jar identical to the one containing the marbles that you guessed the number of before the start of this lab. You also measured the physical dimensions and calculated the volume of a marble identical to the ones contained in the jar.

   a. *GF Use your volume calculations for the jar and an individual marble to estimate how many marbles fit into the jar.*
      
      *List all assumptions you make for this estimation and show all of you work.*

   b. As a class, and with your lab instructor, calculate the average of all the individual guesses for the number of marbles in the jar.
      
      Average of all the Guesses: _______________.

   c. Compare the three values (your individual guess, the average of guesses, and your estimation) for the number of marbles in the jar. Which do you think is the most *accurate* for the true number of marbles in the jar? Explain why you think this considering what you have learned in this lab about measurement, estimation, and the law of large numbers.
### Table 4.2 Student Measurement Calculations

<table>
<thead>
<tr>
<th>Pre-lab Station – How many marbles in jar?</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calculated Estimation</strong></td>
<td><strong>Average of All Student Guesses</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area Calculations</th>
<th>Unit: “Square Strings”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station B – Area of Rectangle</td>
<td></td>
</tr>
<tr>
<td><strong>Station B – Area of Right Triangle</strong></td>
<td></td>
</tr>
<tr>
<td>Station B – Area of Circle</td>
<td></td>
</tr>
<tr>
<td><em>Using radius from diameter</em></td>
<td></td>
</tr>
<tr>
<td><em>Using radius from circumference</em></td>
<td></td>
</tr>
<tr>
<td>Station C – Surface Area of Large Sphere</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume Calculations</th>
<th>Unit: “Cubic Strings”</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Station C – Volume of Large Sphere</strong></td>
<td></td>
</tr>
<tr>
<td>Station C – Volume of Marble</td>
<td></td>
</tr>
<tr>
<td>Station C – Volume of Jar</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle Calculations</th>
<th>Unit: Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Station D – Angle ( \theta ) via trigonometry</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Remote Triangulation &amp; Altitude Calculations</th>
<th>Unit: Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length of the Hallway</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Height of the Projector</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Height of the Fire Detector</strong></td>
<td></td>
</tr>
</tbody>
</table>
16. **Estimation the Number of Galaxies in the Observable Universe**

For this question, you will estimate the number of galaxies in the observable universe. Figure 3 is an image taken by the Hubble Space Telescope called the Hubble Ultra-Deep Field (HUDF). It is a 2.4’ x 2.4’ image of a patch of sky that normally seems devoid of any observable objects. This angular size is roughly equivalent to the amount of sky a penny held 100 feet away from you would cover.

The image combines long-exposure images taken over 6 months with an overall exposure time of just under 1 million seconds (about 11.5 days). Nearly every object in the image is a galaxy, with a total count of over 10,000 galaxies in this small patch of sky. Assuming this number is roughly constant over the entire sky, use this information to estimate the total number of galaxies over the entire sky, which would cover the entire observable universe.

a. There are 360° around the circumference of a circle, and the circumference is related to the radius of a circle by \( \text{Circ.} = 2\pi R \), where \( R \) is the radius of the circle. So, how many degrees are in one radius worth of the circumference? This quantity defines the unit *radian*. Calculate how many degrees are in one radian using \( \text{Circ.} = 360° = 2\pi R \), so one radian is \( R = 360°/(2\pi) \).

b. How many square degrees exist on the entire sky? The surface area of a sphere is \( \text{Surface Area} = 4\pi R^2 \). Use the number of degrees in “one radius” to calculate the total number of square degrees on the sky.
c. The HUDF is 2.4’ x 2.4’ (’ is arc-minute). Convert the arc-minutes to degrees to calculate the number of square degrees on the sky the HUDF covers.

d. Use the total number of square degrees on the sky, and the fact that there are 10,000 galaxies in patch of sky covered by the HUDF to estimate the total number of galaxies in the observable universe.

e. Assume that the count of the number of galaxies in the HUDF is 10,000 +/- 2,500 galaxies (plus or minus 25%). Calculate the high and low estimates for number of galaxies assuming this 25% error in counting the number of galaxies in the HUDF.

f. How does your answer compare to the estimate of a few years ago of 400 billion galaxies in the observable universe?
17. **Uncertainty in Distance from Uncertainty in Parallax**

Consider a far-off star in our galaxy. Using the method of stellar parallax described in Section 1.2.3, you are to measure the star’s parallax to be $0.002 \pm 0.001$ arc-seconds. You feel you have done a great job to get that level of precision in your measurement. Let’s see how that small uncertainty of 0.001 arc-seconds translates to uncertainty in distance.

a. Use the measured value of 0.002 to calculate the distance to the star in parsecs and convert that to light-years (1 parsec = 3.26 light years).

\[ \text{Distance [pc]} = \frac{1}{p[\text{arcsec}]} \]

Distance in parsecs: __________ pc
Distance in light-years: __________ ly

b. Use the measured value minus the uncertainty $0.002 - 0.001 = 0.001$ pc to calculate the distance to the star in parsecs and convert that to light-years (1 parsec = 3.26 light years).

\[ \text{Distance [pc]} = \frac{1}{p[\text{arcsec}]} \]

Distance in parsecs: __________ pc
Distance in light-years: __________ ly

c. Use the measured value plus the uncertainty $0.002 + 0.001 = 0.003$ pc to calculate the distance to the star in parsecs and convert that to light-years (1 parsec = 3.26 light years).

\[ \text{Distance [pc]} = \frac{1}{p[\text{arcsec}]} \]

Distance in parsecs: __________ pc
Distance in light-years: __________ ly

d. Based on your calculations, comment on how well you have determined the distance to the star.
5. Finish All Calculations as Take-Home Work

All questions and calculations you have not yet completed are due at the beginning of next week's lab.