Lab 7: Gravity and Jupiter's Moons

Gravity is the force that binds all astronomical structures. Clusters of galaxies are gravitationally bound into the largest structures in the Universe, Galactic Superclusters. The galaxies themselves are held together by gravity, as are all of the star systems within them. Our own Solar System is a collection of bodies gravitationally bound to our star, Sol. Cutting edge science requires the use of Einstein's General Theory of Relativity to explain gravity. But the interactions of the bodies in our Solar System were understood long before Einstein's time. In chapter two of Chaisson McMillan's *Astronomy Today*, you went over Kepler's Laws. These laws of gravity were made to describe the interactions in our Solar System.

\[ P^2 = \frac{a^3}{M} \]

Where 'P' is the orbital period in Earth years, the time for the body to make one full orbit. 'a' is the length of the orbit's semi-major axis, for nearly circular orbits the orbital radius. 'M' is the total mass of the system in units of Solar Masses.

Jupiter has over 60 moons at the last count, most of which are asteroids and comets captured from...
the Asteroid Belt. When Galileo viewed Jupiter through his early telescope, he noticed only four moons: Io, Europa, Ganymede, and Callisto. The Jupiter System can be thought of as a miniature Solar System, with Jupiter in place of the Sun, and the Galilean moons like planets. In fact it was Galileo's careful observation of the moons orbits around Jupiter that helped astronomers conclude the Earth could not be at the center of the Universe.

This exercise will use Kepler's Third Law of motion to describe the Jupiter System. For this Lab, you will use the free program Celestia to observe the motion of all four Galilean Moons around Jupiter. By measuring their orbital periods, you will be able to find the orbital radius for each moon.

- Mass of Jupiter: $1.89 \times 10^{27}$ kg

Io is the closest Galilean moon to Jupiter and the most geologically active body in the solar system. The tidal forces created by Jupiter's huge gravity and Io's non-circular orbit squeeze and pull the moon, heating it's interior to melt. As a result, Io is covered in volcanoes, there were more than 80 actively erupting volcanoes when the last visiting spacecraft imaged it's surface. The largest volcano, Loki, emits more energy than all of Earth's volcanoes combined and has a larger area than the state of Maryland. The constant volcanic activity continually smooths Io's surface so that is shows neither craters nor streaks.

- Mass of Io: $8.93 \times 10^{22}$ kg

Europa is the second closest Galilean moon to Jupiter, and may have more liquid water than exists on Earth. Europa's surface is a thick layer of water ice lined with cracks and fractures, that covers a saltwater ocean 100km deep. The tidal forces from Jupiter on Europa are responsible for heating up the moon enough to maintain the ocean. The moving tidal bulge is also responsible for the multitude of
cracks on Europa's surface. As the ice cracks, the underlying ice wells up and freezes to the hardness of rock at the surface. Some of the fractures extend halfway around the moon! The presence of liquid water on Europa is a source of intense interest by astronomers looking for life outside of Earth. In fact, when a research vessel around the Jupiter system had completed it's mission, the scientists decided to have the vessel plunge into Jupiter's atmosphere and be crushed by the intense gravity, so that it would not crash to Europa and disrupt any potential life.

- Mass of Europa: $4.80 \times 10^{22}$ kg

Ganymede and Callisto are the third and fourth Galilean moons from Jupiter, respectively. Both Ganymede and Callisto are pockmarked with craters, like the moon. Ganymede is the largest moon in the Solar system, actually bigger than the planet Mercury. The light colored areas on Ganymede's surface are the least cratered, which means they are the youngest areas on the moon. As micro-meteorite dust builds up on the surface, it darkens. So the darker areas are also the oldest. Ganymede separated into layers of a crust, mantle, and core like the Earth during it's formation. Differentiation is the separation of a body into distinct layers while it is molten, so that the densest layer is at the core, and the least dense layer is on top (crust).

Callisto is too small to have been kept warm by it's own radioactivity for long, and too far from Jupiter to experience much tidal heating. So Callisto was not able to differentiate before it solidified during the formation of the Jupiter System. Callisto is more or less just an icy rock with no distinct crust, mantle, or core.

- Mass of Ganymede: $1.48 \times 10^{23}$ kg
- Mass of Callisto: $1.08 \times 10^{23}$ kg

Kepler's Third Law for any system

Kepler's third law as described above is specifically formulated for a planet orbiting the Sun. To use that law for the Jupiter System, we have to generalize it.

$$ P^2 = \frac{4 \cdot \pi^2}{G \cdot (m_1 + m_2)} \cdot a^3 $$

- Where 'P' is the orbital period in seconds
- 'a' is the orbit's semi-major axis in meters
- 'm_1' is the mass of the first object
- 'm_2' is the mass of the second object
- 'G' is the Universal Gravitational Constant equal to $6.67 \times 10^{-11}$ Nm$^2$kg$^{-2}$.

In the case of the Sun-planet system, we change the units of measurement so that all of the extra constants
go to one. But when we use Kepler's laws for any other system, we can't ignore the extra factors and we always use SI units.

In this exercise you will find 'P' and are looking for 'a' with all of the other numbers given. The formula has to be algebraically rearranged to solve for 'a'.

\[ a = \left( \frac{P^2 G (m_1 + m_2)}{4 \cdot \pi^2} \right)^{1/3} \]

Don't let this equation intimidate you, remember you will have all of the numbers you need and are looking for 'a'!

The following are the instructions for using Celestia to find the orbital period of each Galilean moon.

**Instructions for using Celestia**

1. Start Celestia
2. Click on “Navigation” then click on ”Tour Guide”
3. Click the down arrow under “Select Your Destination”.
4. Select Jupiter and click the “Go To” button and then click the “OK” button.
5. Jupiter will appear on your screen.
6. Click Render on the tool bar. Click “View Options…” In the box marked Orbits / Labels make sure ”Moons” and “Planets” are checked in both columns. In the “Information Text” box make sure that “Verbose” is selected. In the upper right hand box (no label) of the View Options window make sure “Orbits and “Ecliptic Line” are checked.
7. You should see something like the following screen:

![Celestia Screen](image)

8. You must rotate Jupiter to orient the planet/moons so you can take your data. With the “Shift” key pressed, press the “down arrow” on the numeric keypad several times until you have rotated Jupiter 90 degrees. You will be looking down on Jupiter's North pole. You should see half the planet in shadow and the other half illuminated by the sun. The screen should appear as follows.
9. In order to use the program to obtain your data, you must zoom out in order to view the Galilean moons as well as Amalthea. Use the wheel on your mouse to zoom out far enough to see the complete orbit of Callisto.

10. Use the “L” key to advance time and the “K” key to slow time.
11. The information you will need is in the upper right corner of the screen. You may use the “Space Bar” to “pause” the motion of the moons in order to record the times.

Once you have found the period for each moon, convert that number so that it has units of seconds, otherwise the formula won't work!

Now insert your 'P' into the general form of Kepler's Third Law.
$a = \left[ \frac{P^2 \cdot G \cdot (m_1 + m_2)}{4 \cdot \pi^2} \right]^{\frac{1}{3}}$

How do your values for 'a' compare to the accepted values of:

- Io 'a' = 4.2*10^5 km
- Europa 'a' = 6.7*10^5 km
- Ganymede 'a' = 1.1*10^6 km
- Callisto 'a' = 1.9*10^6 km
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Mass of Jupiter $= 1.89 \times 10^{27} \text{ kg}$
Mass of Io $= 8.93 \times 10^{22} \text{ kg}$
Mass of Europa $= 4.80 \times 10^{22} \text{ kg}$
Mass of Ganymede $= 1.48 \times 10^{23} \text{ kg}$
Mass of Callisto $= 1.08 \times 10^{23} \text{ kg}$

Universal Gravitational Constant 'G' $= 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

General Kepler's Third Law of Planetary Motion, solved for length of semi-major axis 'a'

$$a = \left( \frac{P^2 \cdot G \cdot (m_1 + m_2)}{4 \cdot \pi^2} \right)^{1/3}$$

1. Period of Io from Celestia, $P_{Io}$
   $$a_{Io} = \left( \frac{P_{Io}^2 \cdot G \cdot (m_J + m_{Io})}{4 \cdot \pi^2} \right)^{1/3}$$
   How does your result compare to $a_{Io} = 4.2 \times 10^5 \text{ km}$?

2. Period of Europa from Celestia, $P_{Eu}$
   $$a_{Eu} = \left( \frac{P_{Eu}^2 \cdot G \cdot (m_J + m_{Eu})}{4 \cdot \pi^2} \right)^{1/3}$$
   How does your result compare to $a_{Eu} = 6.7 \times 10^5 \text{ km}$?

3. Period of Ganymede from Celestia, $P_{Ga}$
   $$a_{Ga} = \left( \frac{P_{Ga}^2 \cdot G \cdot (m_J + m_{Ga})}{4 \cdot \pi^2} \right)^{1/3}$$
   How does your result compare to $a_{Ga} = 1.1 \times 10^6 \text{ km}$?

4. Period of Callisto from Celestia, $P_{Ca}$
   $$a_{Ca} = \left( \frac{P_{Ca}^2 \cdot G \cdot (m_J + m_{Ca})}{4 \cdot \pi^2} \right)^{1/3}$$
   How does your result compare to $a_{Ca} = 1.9 \times 10^6 \text{ km}$?